Brief Outline of Research Findings

The sound field of a structural-acoustic enclosure has been subject to experimental analysis and theoretical description in order to develop an efficient and accurate method for predicting sound pressure levels in enclosures such as aircraft fuselages. Asymptotic Modal Analysis (AMA) is the method under investigation. AMA is derived from classical modal analysis (CMA) by considering the asymptotic limit of the sound pressure level as the number of acoustic and/or structural modes approaches infinity. Using AMA, results identical to those of Statistical Energy Analysis (SEA) have been obtained for the spatially-averaged sound pressure levels in the interior. Reference 1 contains a complete explanation of the AMA method and a formal derivation of the method. AMA is systematically derived from CMA and therefore the degree of generality of the end result can be adjusted through the choice of appropriate simplifying assumptions. For example, AMA can be used to obtain local sound pressure levels at particular points inside the enclosure, or to include the effects of varying the size and/or location of the sound source. AMA theoretical results have been compared with CMA theory and also with experiment for the case where the structural-acoustic enclosure is a rectangular cavity with part of one wall flexible and vibrating, while the rest of the cavity is rigid.

Previous work [Ref. 2, 3, 4] related to the rectangular acoustic cavity has shown that as the number of acoustic modes becomes large, the AMA result approaches the CMA result. The sound pressure levels in the interior are uniform except for the boundaries, where the sound pressure levels are elevated. These boundary regions are referred to as "intensification zones." During this six-month period (October 15, 1989 to April 14, 1990), theoretical research efforts have focused on the intensification zones. Locally, i.e. in the intensification zone, the cavity problem can be modeled as an infinite number of equal strength random incidence sound waves striking a wall (or a corner, or an edge) and being reflected. Only the wall (or corner, or edge) is included in this model, the other dimensions extend to infinity, as if there were no cavity. The AMA result describing the intensification zone was also independent of cavity dimensions. Summing the contributions of each sound wave was achieved by integrating over all possible angles. This result was identical to that predicted by AMA, further verifying the method. The case of a non-rigid wall can be solved by applying an impedance at the boundary and proceeding as before by integrating over the incidence angles. This insight into the intensification zones will allow further development of the AMA method near absorptive walls, for example when a wall or walls are covered with acoustic foam.

The sound pressure levels within an intensification zone and its thickness were found to be independent of the dimensions of the cavity. In fact, the variation of sound pressure level with distance into the cavity depends only upon the bandwidth to center frequency ratio, if the distance into the cavity is non-dimensionalized by the center frequency wavenumber. During the six-month time period covered by this report, the separate effects of bandwidth and center frequency were studied. By expanding the AMA expression for mean square pressure in a Taylor Series about the center frequency wavenumber the effect of bandwidth can be separated from the center frequency effect. A new expression is obtained, which consists of a function of center frequency only plus a second order term which contains the bandwidth effect. [The term which is linear in bandwidth evaluates to zero, using the arithmetic definition of center frequency]. The higher order terms are at least fourth order in bandwidth. Therefore, a universal curve can be constructed for a given trajectory into the cavity for a given center frequency, and the bandwidth effects can be added as a correction.

Both the study of the bandwidth effects and the comparison between AMA in the intensification zone using a model of oblique incidence sound waves impinging on a rigid (or absorptive) wall were presented at the 118th Acoustical Society of America Meeting in St. Louis, Mo. November 27- December 1, 1989. A copy of the viewgraphs is attached to this progress report.

Experiments were performed at NASA Langley Research Center throughout October 1989 and again throughout February 1990. The main goals of the experiments are to provide data with which to verify the AMA method and to study the intensification zone behavior near walls, edges and corners of the rectangular acoustic cavity. A few preliminary experimental results were presented at the ASA meeting in St. Louis (see attached viewgraphs). Early results show good agreement between experiment and theory. The interior region appears uniform, while sound pressure levels are elevated at the corners, edges and walls. As predicted, the levels are approximately 9, 6 and 3 dB higher at the corners, edges, and walls than they are in the interior. Agreement is best for certain choices of center frequency and bandwidth, although the assessment is not complete at this time. Experimental data is still being processed, and a paper reporting the experimental findings is in progress.

References

1. Dowell, E. H., "Vibration Induced Noise in Aircraft: Asymptotic Modal Analysis and Statistical Energy Analysis of Dynamical systems," R82-112447, United Technologies Research Center, January 1983.

2. Kubota, Y., Dionne, H.D., and Dowell, E.H., "Asymptotic Modal Analysis and Statistical Energy Analysis of an Acoustic Cavity," Journal of Vibration, Acoustics, Stress, and Reliability in Design, Vol. 110, No. 3, 1988, pp. 371-376.

3. Peretti, L., "Asymptotic Modal Analysis of a Rectangular Acoustic Cavity," MSE Thesis, Duke University, 1988.

4. Peretti, L. and Dowell, E.H., "Asymptotic Modal Analysis of a Rectangular Acoustic Cavity and Characterization of its Intensification Zones," AIAA Paper 89-1073, AIAA 12th Aeroacoustics Conference, San Antonio, TX, April, 1989.

5. Peretti, L.F. and Earl H. Dowell, "Bandwidth and Absorption Effects on Intensification in a Structural-Acoustic Enclosure," presentation at 118th Acoustical Society of America Meeting, 27 November - 1 December 1989.

Viewgraphs:

Presentation at the 118th Acoustical Society of America Meeting
St. Louis, Mo.   27 November - 1 December 1989

Bandwidth and Absorption Effects on Intensification in a Structural-Acoustic Enclosure
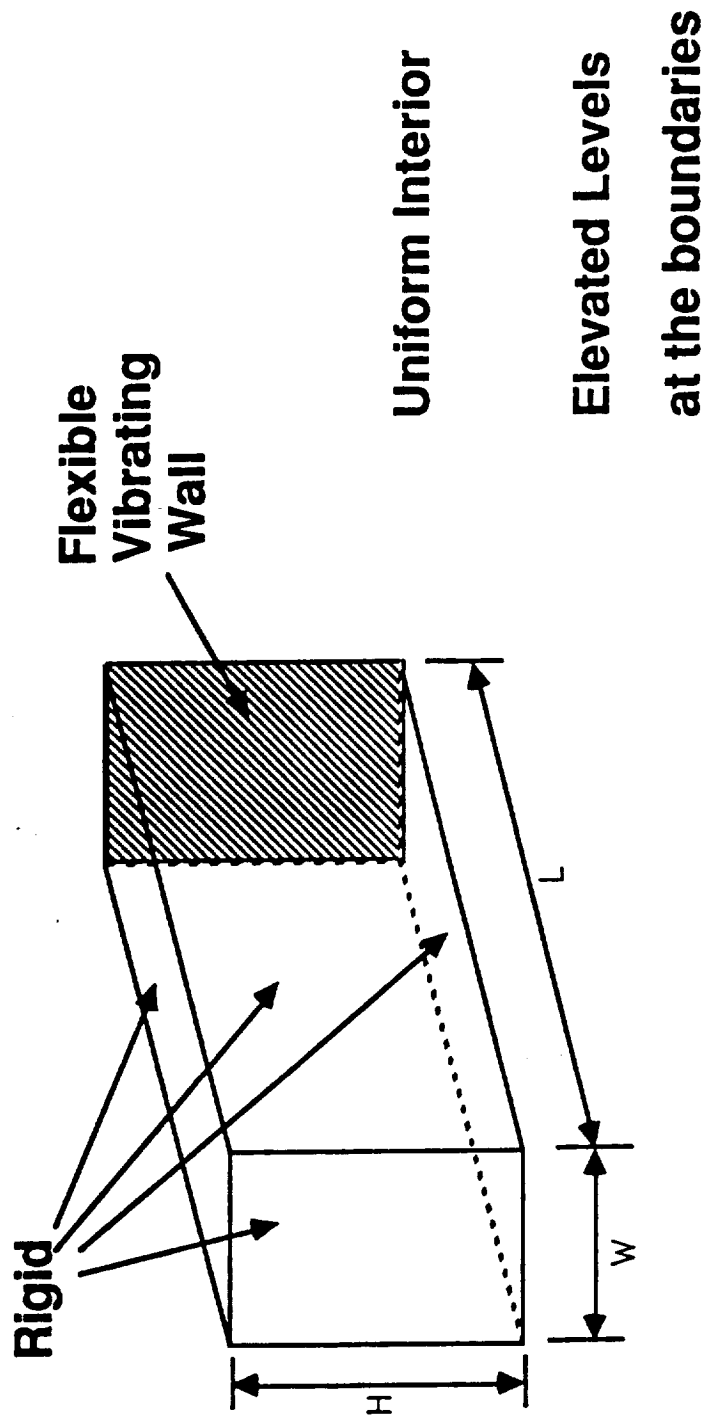
Linda F. Peretti
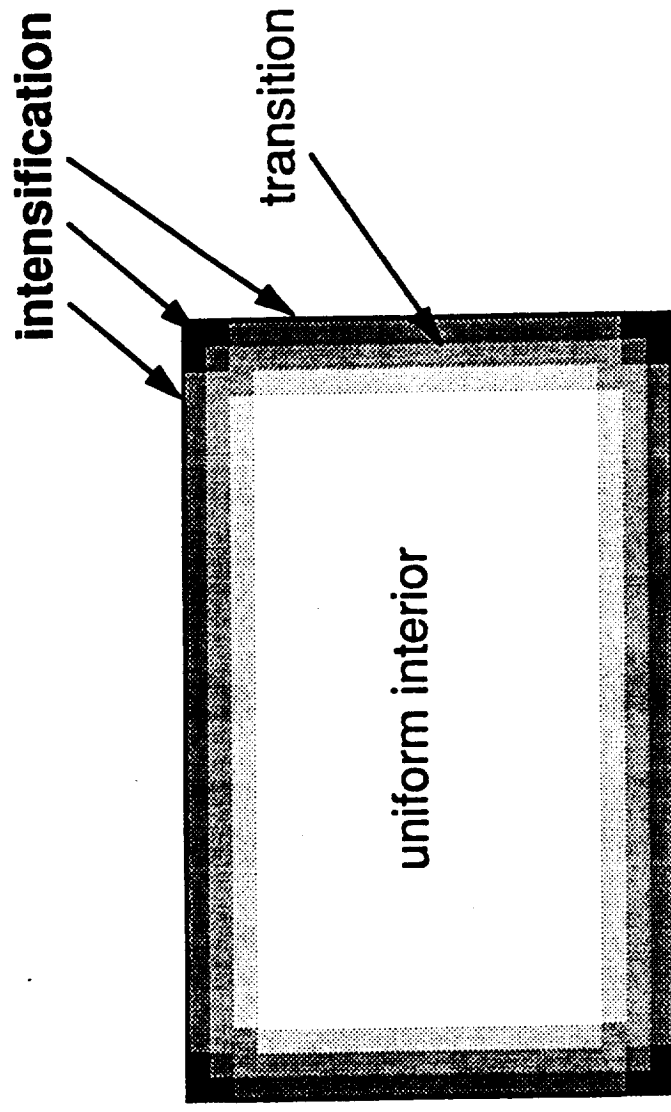
Earl H. Dowell

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Duke University

Asymptotic Modal Analysis [AMA]

Classical Modal Analysis $\rightarrow$ Statistical Energy Analysis

Large Number of Modes

Flexible Vibrating Wall

Rigid

L

W

H

Uniform Interior

Elevated Levels at the boundaries

Intensification

intensification

intensification

transition

uniform interior

- **Physical Mechanism (correlated modes vs. uncorrelated modes)**
- **Mathematical Boundary Layer**
- **Experiment Versus Theory**

Asymptotic Modal Analysis Result for 3-D Waves
kcx = kcy = kcz
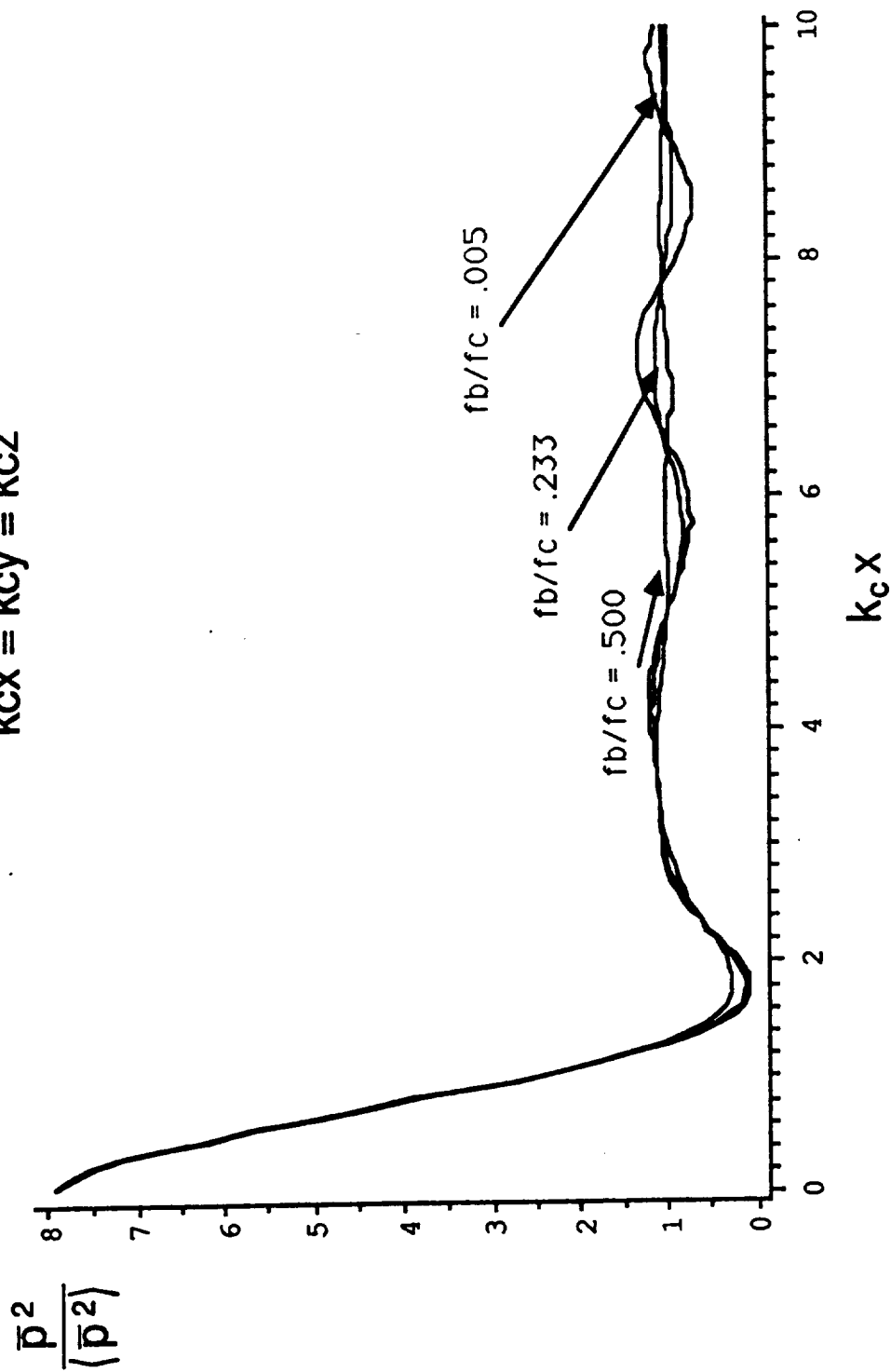
AMA Result For 3-D Mode From A Wall

For 3 Bandwidth/Center Frequency Ratios

increasing bandwidth

$\dfrac{\overline{p^2}}{\langle \overline{p^2} \rangle}$

$k_c x$
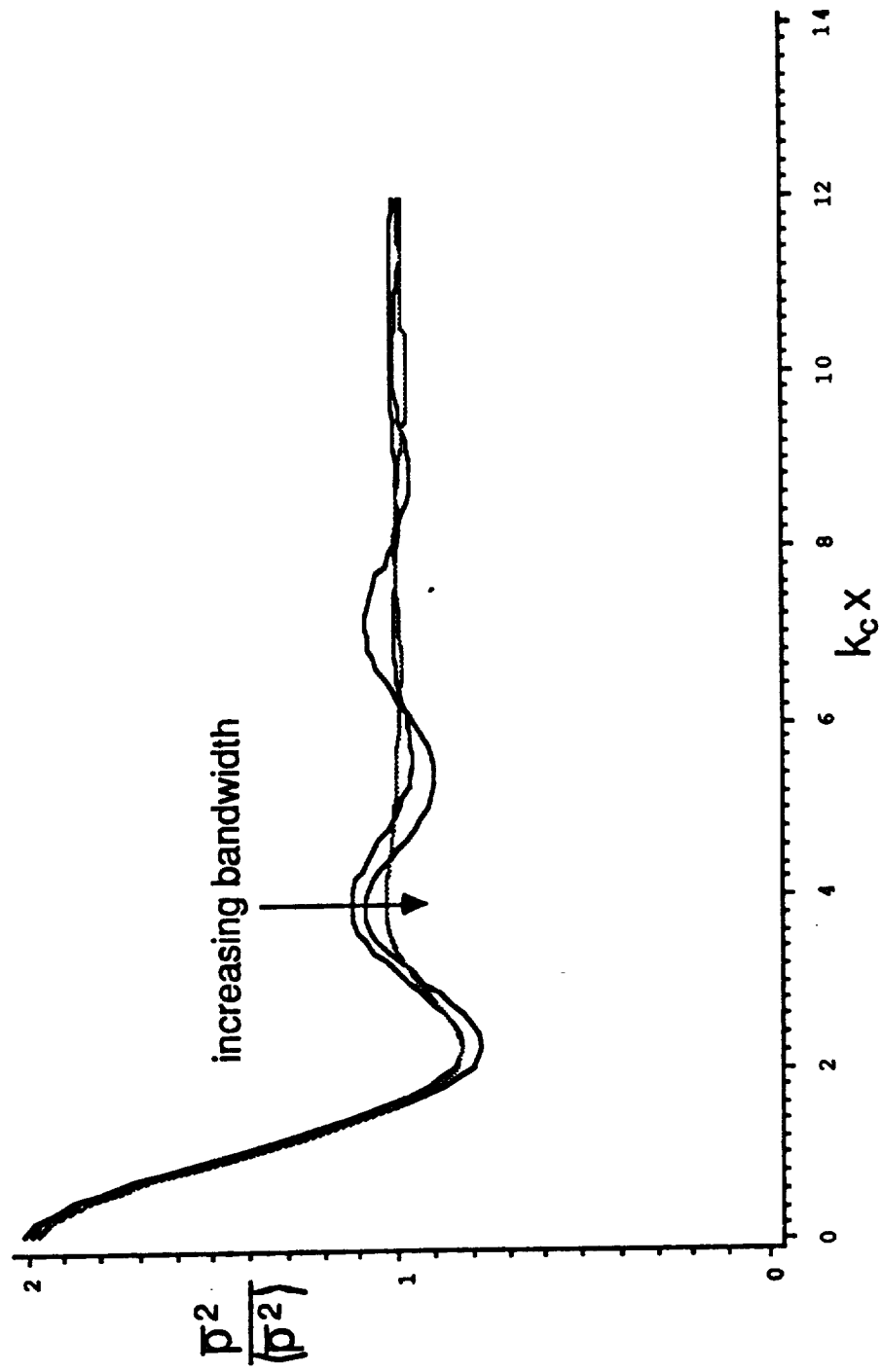
Locally, Intensification looks like Oblique Waves

$$\overline{p}^2 = 2\,|A|^2 \iint \cos^2(k\,x\cos\theta)\,d\theta\,dk$$
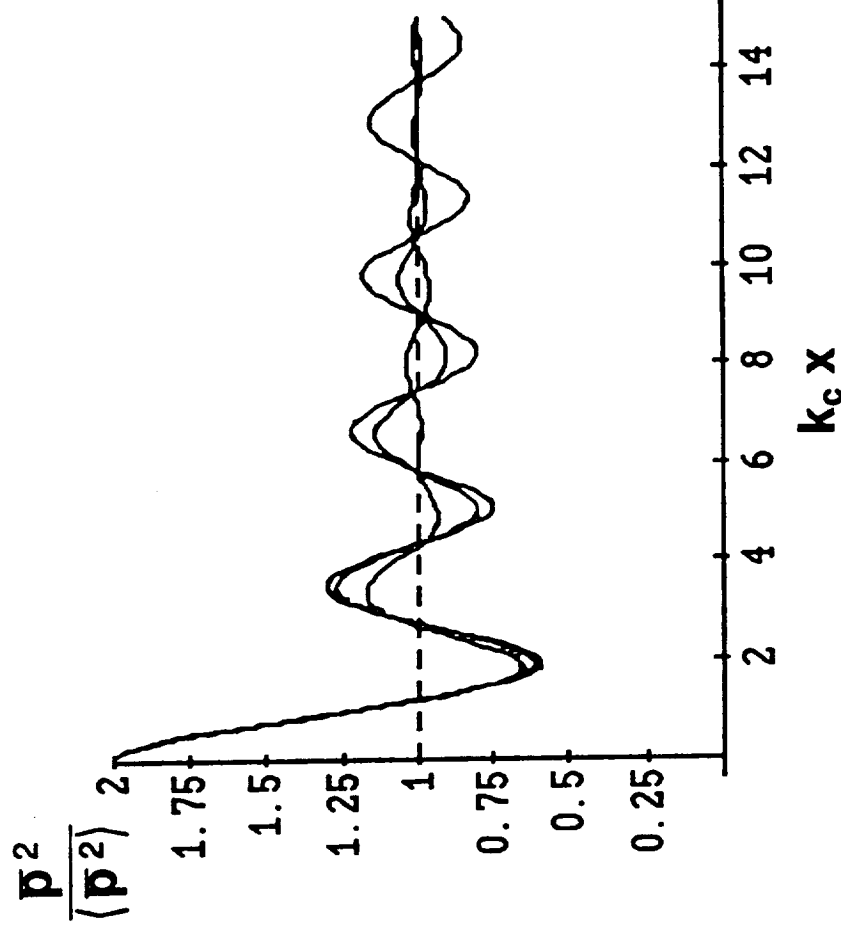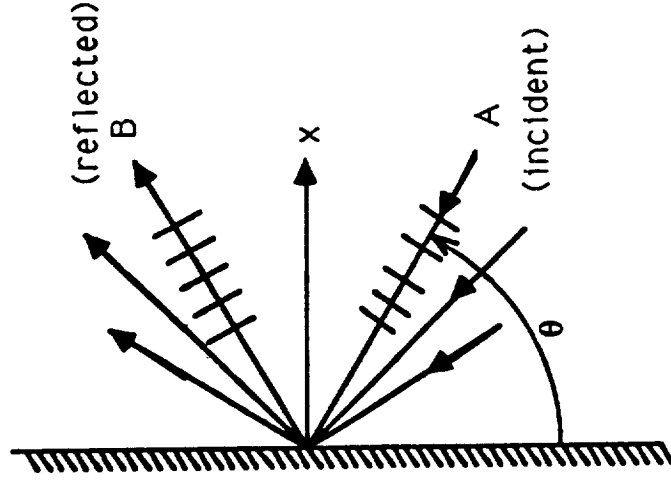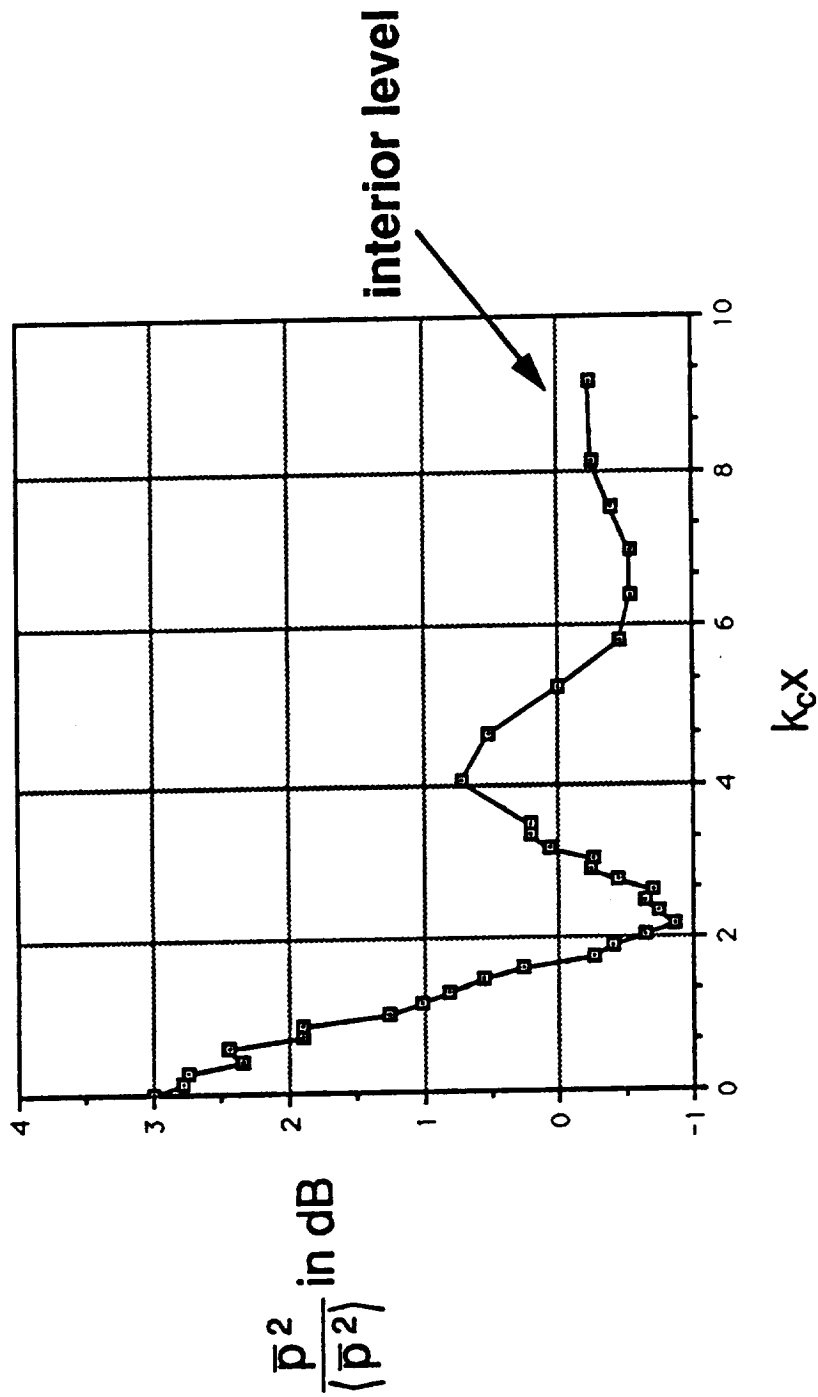

Curve for 3 bandwidth/center frequency ratios

fb/fc = .005, fb/fc = .2329, fb/fc = .500

Experiment

Flexible
Vibrating
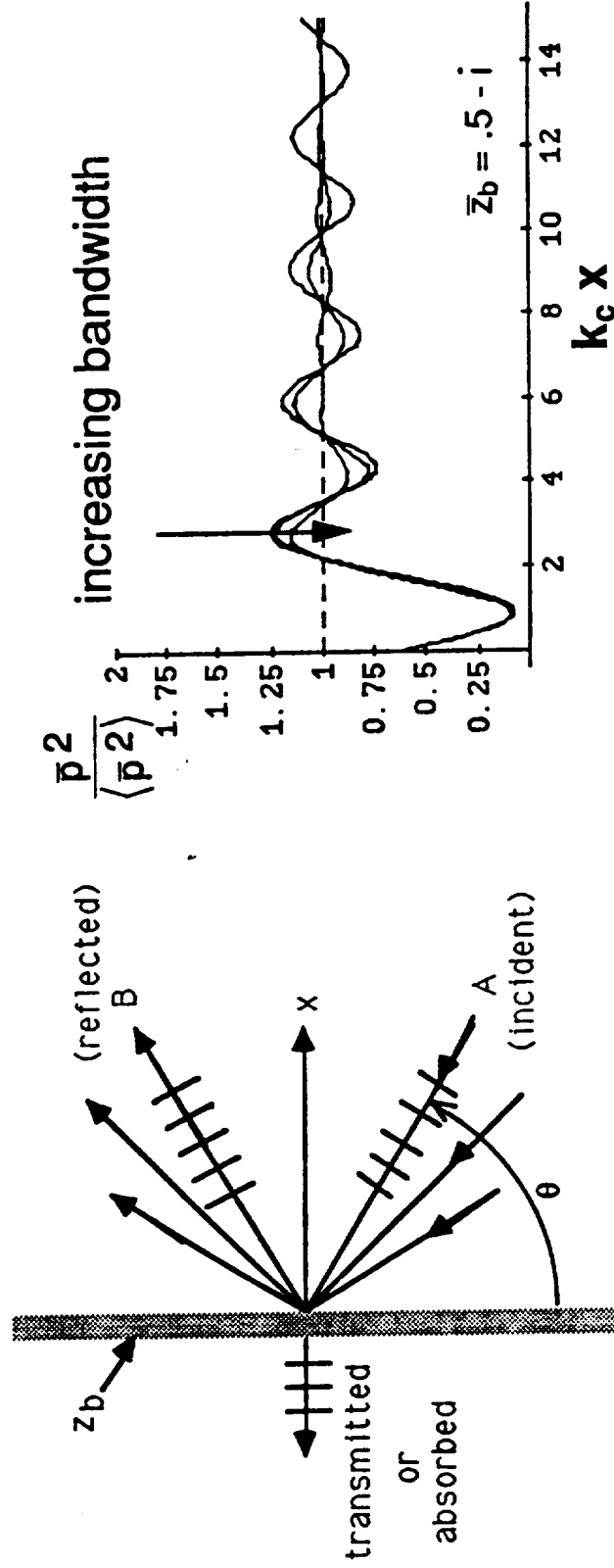Portion
(Sound Source)

Microphone probe

3'

2'

7'
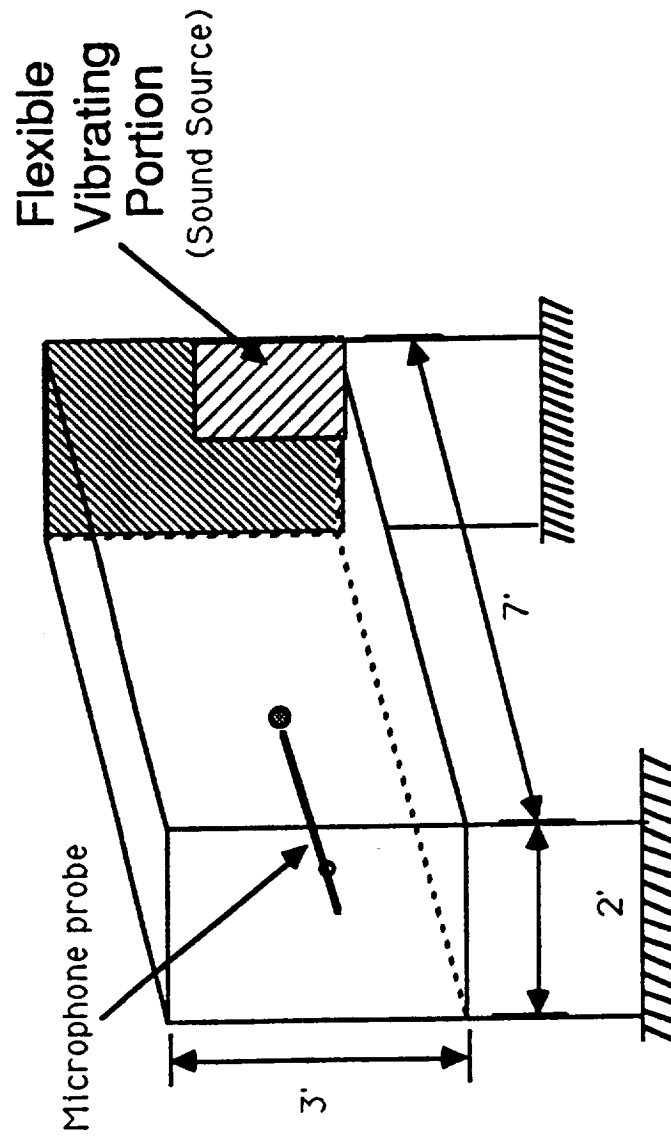
Rigid Wall Experimental Data

Experimental Data 1/3 octave band
Center frequency = 2500 hz

interior level
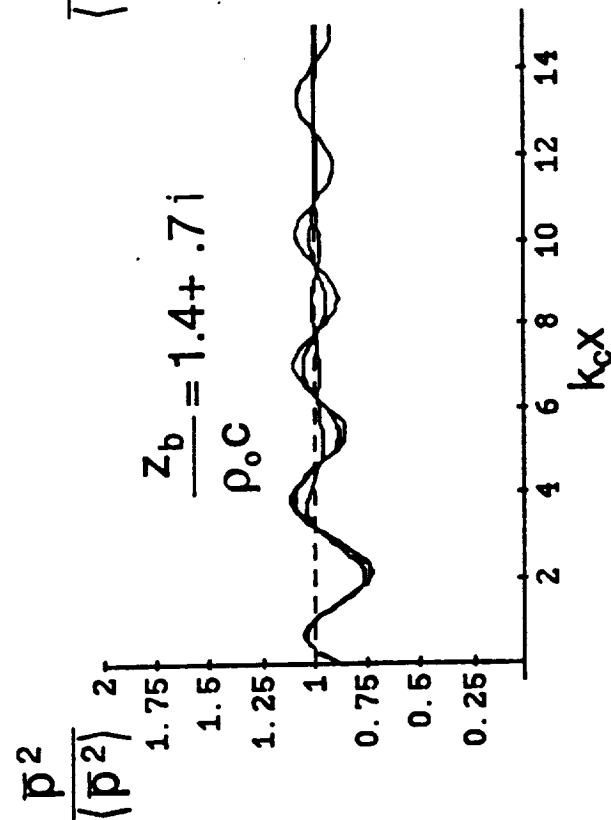
$\dfrac{\overline{p}^2}{\langle \overline{p}^2 \rangle}$ in dB

$k_c x$

Absorption Effects

$$\bar{p}^2 = 2|A|^2 \int\int \left[\cos^2(kx\cos\theta) - 2\cos(kx\cos\theta)\times \frac{(r_b\cos\theta+1)\cos(kx\cos\theta)-x_b\cos\theta\sin(kx\cos\theta)}{(r_b\cos\theta+1)^2+(x_b\cos\theta)^2} + \frac{1}{(r_b\cos\theta+1)^2+(x_b\cos\theta)^2} \right] d\theta\, dk$$

increasing bandwidth

$\dfrac{\bar{p}^2}{\langle\bar{p}^2\rangle}$

2, 1.75, 1.5, 1.25, 1, 0.75, 0.5, 0.25

$k_c\, x$: 2, 4, 6, 8, 10, 12, 14

$\bar{z}_b = .5 - i$

B (reflected)

x

A (incident)

θ

z_b

transmitted or absorbed
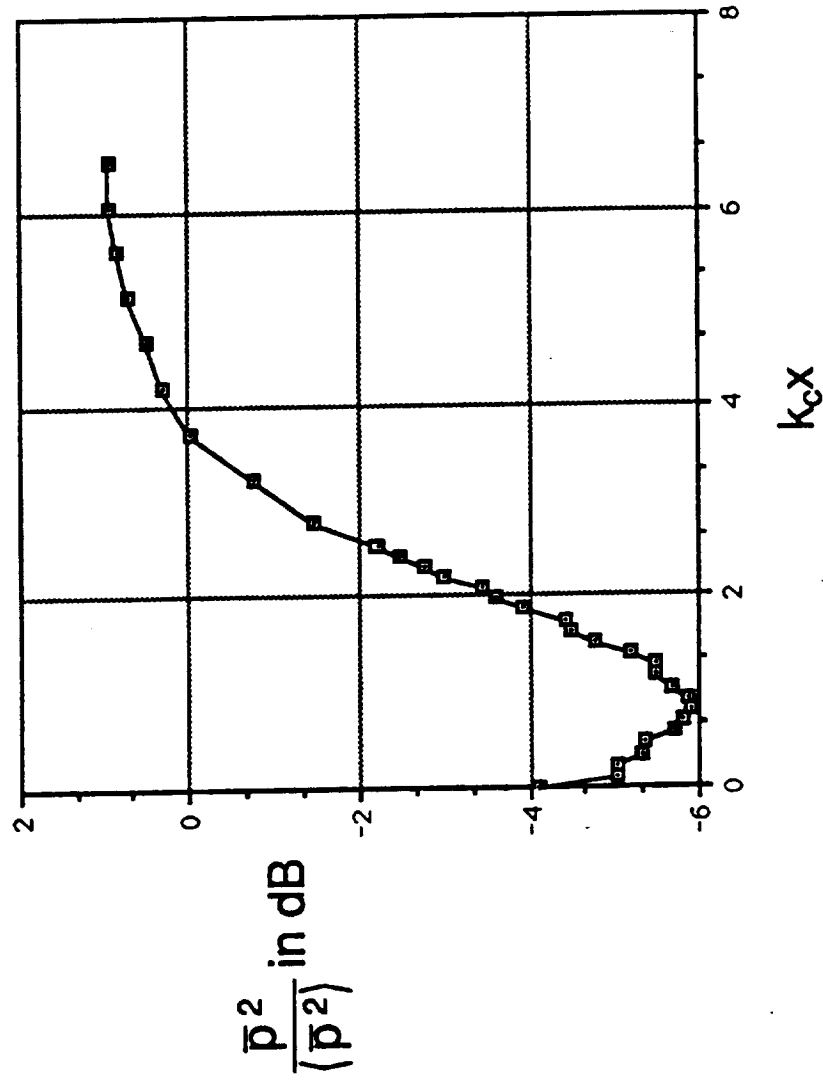
Curve for 3 bandwidth/center frequency ratios
fb/fc = .005, fb/fc = .2329, fb/fc = .500

Theoretical Data with Absorption

$$\frac{z_b}{\rho_o c} = 1.4 - 5.0 i$$

$$\frac{z_b}{\rho_o c} = 1.4 + .7 i$$

$$\frac{z_b}{\rho_o c} = 1.0 + 0 i$$

$\frac{\overline{p}^2}{\langle \overline{p}^2 \rangle}$

$k_c x$
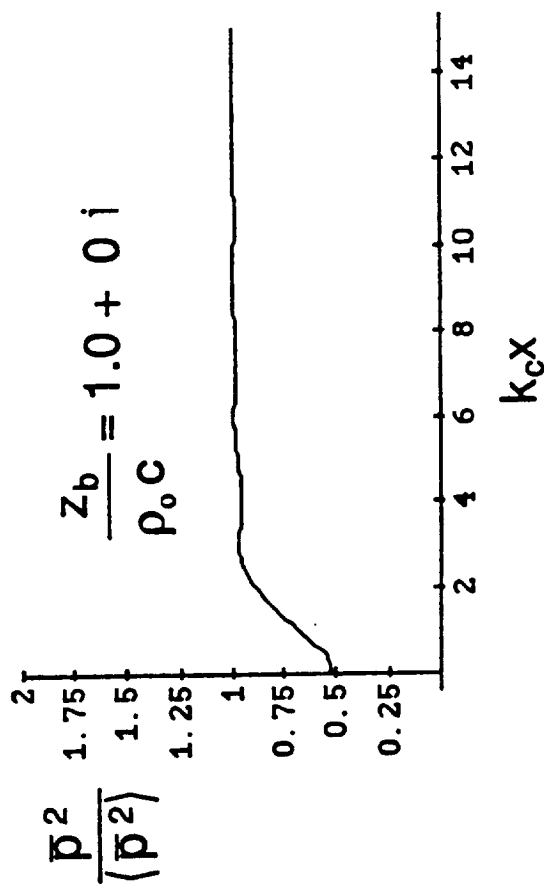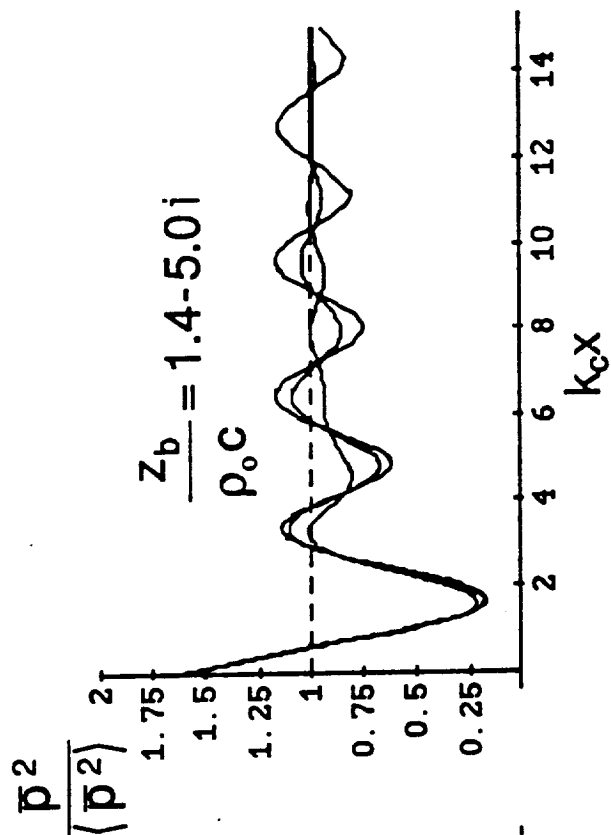
Experiment with Absorption

Experimental Data
Foam on Wall, 1/3 octave band, 2000 hz

Separation of Bandwidth Effects

$$\bar{p}^2 = C \int_{k_\ell}^{k_u}\int_0^{\pi/2}\int_0^{\pi/2} \cos^2(kx\sin\theta\cos\phi)\cos^2(ky\sin\theta\sin\phi)\cos^2(kz\cos\theta)\, k^2\sin\theta\, d\theta\, d\phi\, dk$$

$$\bar{p}^{-2} = \iiint f(k,\theta,\phi)\, d\theta\, d\phi\, dk.$$

$$\bar{p}^{-2} = \int_{k_\ell}^{k_u} f(k)\, dk.$$

$$\bar{p}^{-2} = \int_{k_\ell}^{k_u} \tilde{f}(k_c) + (k-k_c)\tilde{f}'(k_c) + \frac{(k-k_c)^2}{2!}\tilde{f}''(k_c) + \ldots\, dk$$

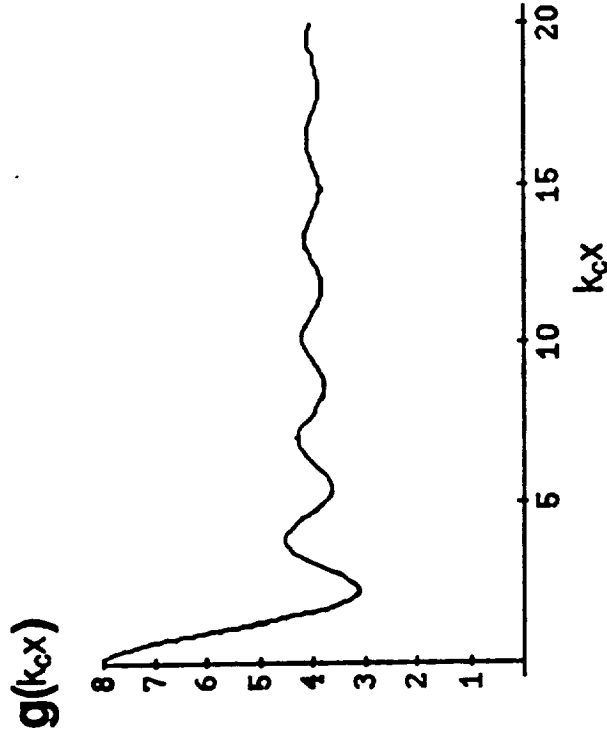## For Arithmetic center frequency odd powers integrate to zero

$$\bar{p}^{-2} = \tilde{f}(k_c)\Delta k + \frac{\Delta k^3}{3\times 2!}\tilde{f}''(k_c) + \ldots$$

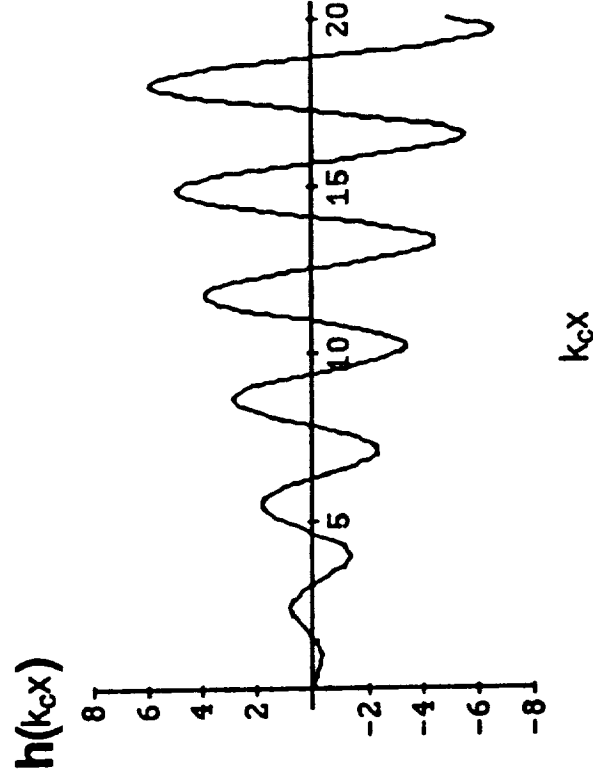## Non-dimensionalizing with spatially averaged value

$$\frac{\bar{p}^{-2}}{<\bar{p}^{-2}>} = g(k_c x) + \Delta k^2\, h(k_c x) + O(\Delta k^4) + \ldots$$

Bandwidth Effects - Universal Curves

$h(k_c x)$

$k_c x$

bandwidth correction

pre-multiply this curve

by (kb/kc) squared

$g(k_c x)$

$k_c x$

Curve for no bandwidth

Bandwidth Effect

solid line = exact solution

dotted line = approximate solution (2 terms in Taylor series)

bandwidth/center frequency = .5
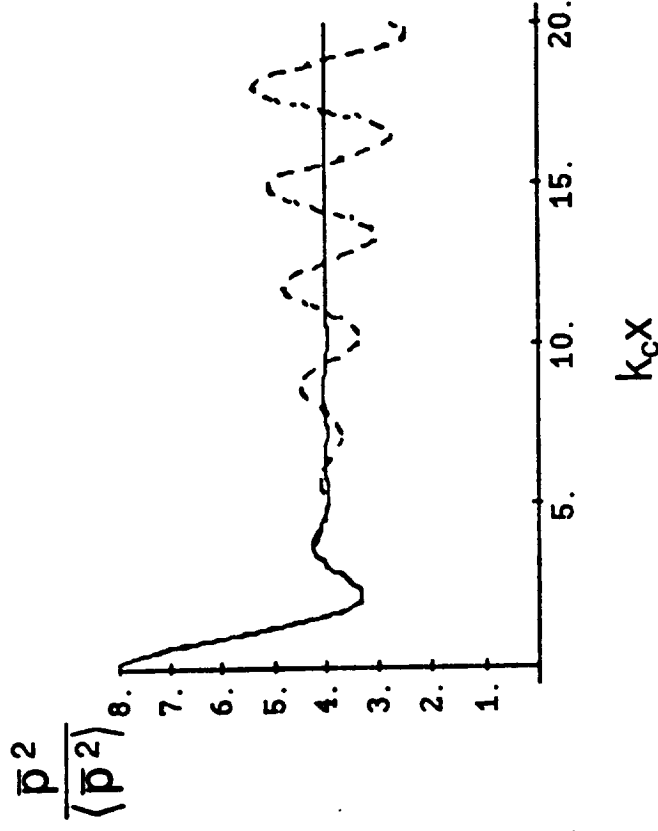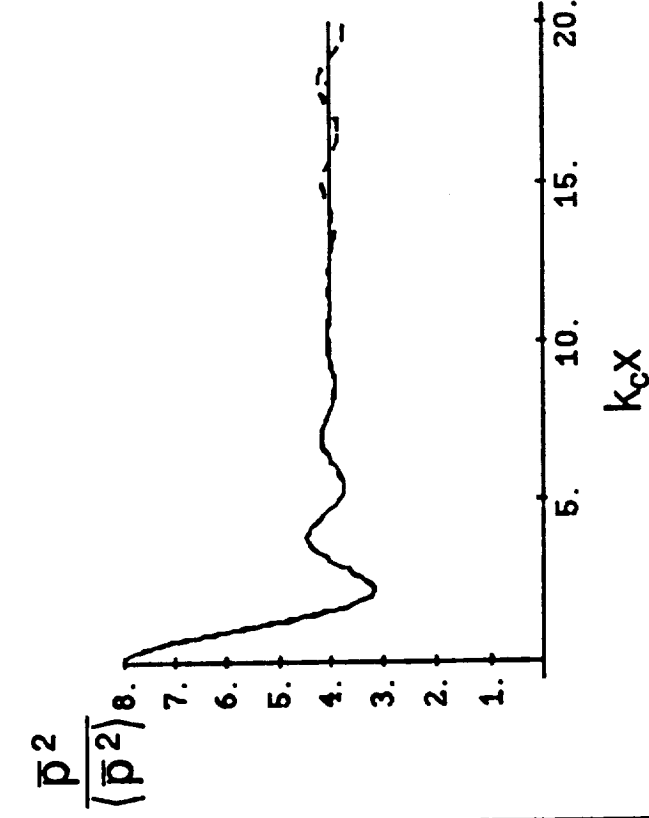
1/3 octave exact vs. approx

On-Going Efforts

Further Experimental Studies:

° Intensification curves coming away from a corner & an edge at arbitrary angles into the cavity

° Intensification curves for alternative sound source configurations -- panel size and location

Further Theoretical Studies:

° AMA solution for case where the vibrating panel contains finite number of structural modes

° Extension of work presented herein, regarding absorption effects and bandwidth effect separation

Conclusions

° AMA method gives same results for an intensification zone of an acoustic cavity as random incidence oblique waves

° The result for non-dimensional sound pressure level can be written in terms of a function of center frequency plus a bandwidth correction factor

° Absorption on a hard wall has the effect of shifting the maximum away from the wall and changing the levels of the peaks. An absorption term can also be separated out of the expression for non-dimensionalized mean-square pressure.

Universal Curve g(kcx) for 2-D modes and kcx = kcy
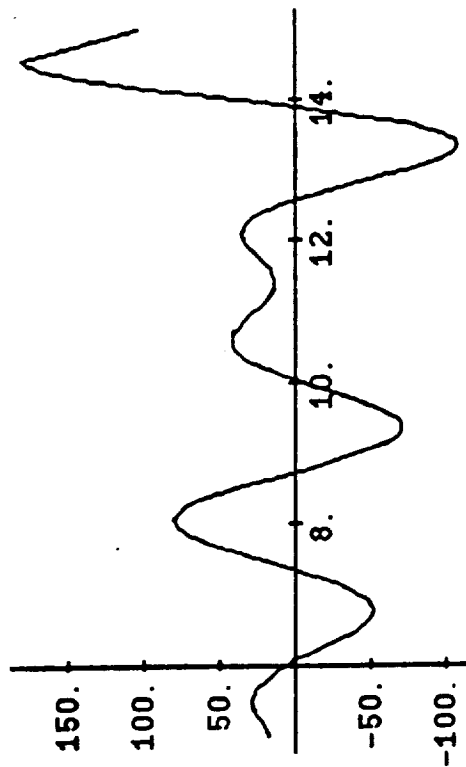
Universal Curve h(kcx) for 2-D modes and kcx=kcy
need to multiply by (kb/kc) squared and add to above